
There are some non-trivial problems in quantization of gauge fields because of the gauge invariance.

Take QED for example:

Even at the classical level a solution of the euler-lagrange equation of motion for the vector potential in terms of conserved current does not exist due to gauge invariance. This is because the co-efficient of the quadratic term in the fields(the inverse of which is propagator or green's function) does not have the inverse as its determinant vanishes. From another point of view, this coefficient is actually a projection operator projecting only transverse modes and hence does not have an inverse.

So we are obliged to choose a gauge.

But once we choose a gauge the lagrangian is no longer invariant under the gauge transformation. But we know that QED current is conserved and it follows from the gauge invariance of the lagragian. This looks like some contradiction. The point is that current conservation follows from the global gauge invariance which is preserved even after one chooses a gauge. Choice of gauge only spoils the local gauge invariance and hence does not affect the current conservation. This means, the choice of gauge should be such as not to spoil the global gauge invariance.

So far this is the problem at the classical level which can be evaded by choice of an appropriate gauge. At the quantum level the problem is compound.

Cannonical quantization: In this quantization one postulates cannonical commutation relations between the field variables and the coresponding conjugate momentum. Here it turns out that momentum conjugate to A^0(time component of Vector potential A) vanishes and hence it is not possible to define a covariant commutation relations. By straight forward differentiation one can show that such commutation relations contradict the Maxwell equations. So either change the commutation relations or change the maxwell equations. Both the options are possible.

Path-Integral quantization: Here also due to Gauge-Invariance one runs into problems. The functional integral is badly divergent because we redundantly integrate over a continuous infinity of physically equivalent field configurations. To fix the problem one should isolate the interesting part of the functional integral which counts each physical configuration only once. This is best done with Path integral method. Infact for non-abelian field(as we will discuss) this is the only way one can tackle the problem of gauge fixing.

In cannonical quantization one fixes the gauge in advance. The power of path-integral lies in the fact that you can choose whatever gauge you want since here gauge fixing is performed by inserting certain delta function into the path-integral. We can change the gauge by simply replacing this factor(the Fadeev-Popov method).

Historically, however before FP, Feynman showed that naive quantization of Yang-Mills theory is non-unitary. To cancel these non-unitary terms Feynman concocted extra terms which just cancel these non-unitary contributions to restore unitarity. Since these extra terms do not follow from quantization procedure they were termed as `ghost'. Today we know them as Fadeev-Popov ghost which follow naturally from the theory when gauge fixing is done properly(using a brilliant trick).

Moral of the story is that you **cannot quantize without fixing the gauge**. Gauge fixing is best done in path-integral method. What one does is that one introduces a delta function of the gauge fixing condition (say for example lorentz gauge) explicitly in the path-integral. That is the path-intgral makes sense only when the gauge-fixing condition is sastisfied, thus forcing the gauge fixing. But how can you put delta function(rather functional) arbitraily in the path integral. So one inserts an `identity'(you can always insert `1') such that it can be written in terms of an integral over all the gauge transformations and a delta functional of the gauge-fixing condition. Such a quantity is certainly not a unity as it changes the measure of integration. So to make it unity you multiply it by a term popularly known as Fadeev-Popov term. This term turns out to be a determinant and hence fadeev-popov determinant.

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